Section 1.6
Rational Exponents

Negative Exponent

If a is a nonzero number and n is any integer, then

$$a^{-n} = \frac{1}{a^n}$$

Quotient Rule

For all integers m and n and all nonzero real numbers a,

$$\frac{a^m}{a^n} = a^{m-n}$$

$a^{1/n}, n$ Even If n is an even positive integer, and if $a > 0$, then $a^{1/n}$ is the positive real number whose nth power is a. That is, $\left(a^{1/n}\right)^n = a$. In this case, $a^{1/n}$ is the principal nth root of a.

$a^{1/n}, n$ Odd If n is an odd positive integer, an a is any real number, then $a^{1/n}$ is the positive or negative real number whose nth power is a. That is, $\left(a^{1/n}\right)^n = a$.

Rational Exponent

For all integers m, all positive integers n, and all real numbers a for which $a^{1/n}$ is a real number:

$$a^{m/n} = \left(a^{1/n}\right)^m .$$

Definitions and Rules for Exponents

Let r and s be rational numbers. The results here are valid for all positive numbers a and b.

$$a^r \cdot a^s = a^{r+s} \qquad (ab)^r = a^r \cdot b^r \qquad (a^r)^s = a^{rs}$$

$$\frac{a^r}{a^s} = a^{r-s} \qquad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r} \qquad a^{-r} = \frac{1}{a^r}$$

Perform each operation mentally.

1) $\dfrac{3.3^3}{1.1^3} = 3^3 = 27$

$\left(\dfrac{3.3}{1.1}\right)^3$

$\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$

2) $(.25^2)(44^2)$

$(.25 \cdot 44)^2$

11^2

121

If $x^2 = 20$, what is x^6 ?

$x^6 = (x^2)^3$

$(x^2)^3 = (20)^3$

$x^6 = 8000$

Simplify using the rules of exponents (only + exponents)

1) $\left(x^{\frac{3}{2}}y^{\frac{1}{5}}\right)^{10} = x^{\frac{3}{2}\cdot 10}y^{\frac{1}{5}\cdot 10} = x^{15}y^{2}$

2) $\left(y^{z+2}\right)^{4} = y^{4(z+2)} = y^{4z+8}$

3) $\dfrac{(x-1)^5}{(x-1)^3} = (x-1)^2$

$4^{1/2} = \sqrt{4} = 2$

4) $\left(\dfrac{4xy^5z^2}{x^{-2}yz^3}\right)^{\frac{1}{2}} = \left(\dfrac{4x^3y^4}{z}\right)^{1/2} = \dfrac{4^{1/2}x^{3/2}y^2}{z^{1/2}} = \dfrac{2x^{3/2}y^2}{z^{1/2}}$

$\dfrac{x^1}{x^2} = x^3$

5) $\dfrac{2^{\frac{1}{4}} \cdot 2^{\frac{5}{4}}}{2^4} = \dfrac{2^{3/2}}{2^4} \dfrac{1}{2^{5/2}}$

$-2\frac{1}{2} = -5/2$

6) $\dfrac{(x^3)^{y+2}}{2x^y}$ where $y < -3$

$\dfrac{x^{3y+6}}{2x^y} = \dfrac{x^{2y+6}}{2} = \dfrac{1}{2x^{-(2y+6)}} = \dfrac{1}{2x^{-2y-6}}$

7) $\dfrac{(x^3)^{y+2}}{2x}$ where $y > -2$

$$\frac{x^{3y+6}}{2x^1} = \frac{x^{3y+5}}{2}$$

$$3y + (6 - 1) = 3y + 5$$

Homework:

Day 4 Assignments